E1 - Magnetic Pendulum (10 pts)

The oscillation frequency of a pendulum can be modified by magnetic forces between the pendulum and its support. In this experiment you will study pendulum motion in a combined potential from gravitational and magnetic interaction terms, using the setup shown in Fig. 3.

Equipment (see also Fig. 3)

- A Pendulum body with point-like supports and mirror for angle measurement
- B Pendulum tower with hard points to support the pendulum, and laser module for angle measurement
- C Rails to support external magnets
- D 2 small dipole magnets to be attached to pendulum body (may be **green, red, white** or **yellow**)
- E 2 identical external dipole magnets (**black**)
- F 2 unknown external magnets F1, F2 (**blue**, F2 is marked with **white** dots at its ends)
- G Screen for laser spot for angle measurement
- H Stopwatch
- I Masking tape, e.g. to fasten pendulum tower to table
- J Pencil and ruler

The magnets are quite strong. Be careful not to hurt yourself or damage the magnets.

Do not look directly into the laser beam, and turn the laser off when not needed.

When experimenting with the pendulum, make sure the supporting screws are resting in the grooves on the pendulum tower.

Feel free to mark the pendulum with your pencil if needed.

Task E1.1 - Masses (1.0 pts)

The total mass of the pendulum body with attached small dipole magnets is $M_{\text{pen}} + M_{\text{mag}} = (52.3 \pm 0.2)\,\text{g}$.

Determine both M_{pen} and M_{mag} as accurately as possible.

Task E1.2 - Magnetic dipole moments (4.0 pts)

With external magnets nearby, the magnetic pendulum moves in a combined potential formed by gravity and magnetic interaction. The resulting pendulum frequency ω can be written as a function of natural frequency ω_1 and "magnetic frequency shift" ω_{mag}:

$$\omega^2 = \omega_1^2 \pm \omega_{\text{mag}}^2 \qquad (1)$$

For the case of two **black** external dipole magnets, symmetrically placed at a distance d around the pendulum equilibrium position (see Fig. 1), and small amplitude oscillations the magnetic frequency shift is:

$$\omega_{\text{mag}}^2 = \frac{6\mu_0}{I\pi} \cdot j_1 \cdot j_2 \cdot \frac{\ell^2}{d^5}, \qquad (2)$$

where $\mu_0 = 4\,\pi \cdot 10^{-7}\,\text{N/A}^2$ is the permeability of vacuum, I is the moment of inertia of the magnetic pendulum around the axis of rotation, j_1 is the combined magnetic moment of the pendulum magnets, j_2 is the magnetic moment of each external dipole, and ℓ is the distance of the pendulum magnet to the rotation axis. For the relative strength of the dipole moments you may assume $j_2 = 2.4 \cdot j_1$. Local gravity is $g = 9.81\,\text{m/s}^2$.

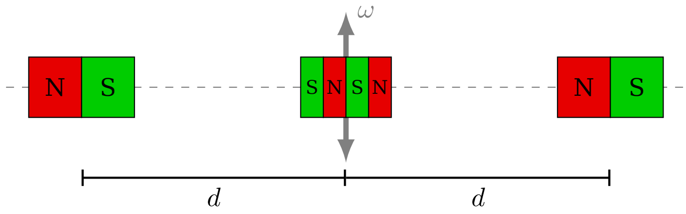

Figure 1: Frequency shifting using external dipole magnets (top view). d denotes the distance between magnet centers. Note that the orientation of the external magnets may be reversed.

a) Measure the pendulum frequencies for different magnet distances d, using very small amplitudes. Make sure to cover the whole accessible frequency range.

b) Determine the "average magnetization" (magnetic moment per unit mass) of the material of pendulum magnets and external dipole magnets. Create a relevant graph for your analysis. Auxiliary measurements may be necessary to determine all unknowns. You may neglect the mass and thickness of the non-magnetic coating of the magnets.

Precise alignment of the rails is important. Make sure that, with the pendulum in its equilibrium position, the centers of all magnets are on a single line.

Make sure to use symmetric configurations to cancel the force on the pendulum magnets along the direction of the rails.

Task E1.3 - Unknown external magnets (3.0 pts)

The two **blue** unknown external magnets (F1, F2) each contain several magnetic dipoles. The dipoles inside F1 are reversed with respect to those inside F2. The magnetic frequency shift in a setup analogous to Fig. 1 also follows a power law:

$$\omega_{\text{mag,F}}^2 \propto d^{\alpha}. \qquad (3)$$

a) Measure the pendulum frequencies for different distances d, using very small amplitudes. Choose settings that allow finding the magnetic frequency shift as accurately as possible.

b) Determine the power law exponent α.

c) Sketch a possible configuration of magnetic dipoles inside F1 and F2 and justify your choice.

Task E1.4 - Nonlinear pendulum (2.0 pts)

Return the setup to the configuration used in Task E1.2, with **black** external dipole magnets arranged as in Fig. 1. Following Eqn. 1, the small-amplitude pendulum frequency can be fully cancelled, $\omega \to 0$.

a) Determine as accurately as possible the magnet separation d required for this full cancellation.

b) Investigate the dependence of pendulum period on its amplitude when tuned to the best cancellation you were able to obtain.

Suggest a functional dependence and validate it with your data.

Discuss the origin of any possible mismatch.

E2 - Optical black-box (10 pts)

WARNING!

Do not open the black-box. Do not shake the black-box. Do not touch the windows of the optical ports. If you break the black-box or the windows or attempt to open the black box, you will be disqualified.

Your task is to determine the contents of an optical black box without opening it.

The black-box has four optical ports (A, B, C and D) for light, and two optical axes (Fig. 2). The optical axes are perpendicular to each other. There is up to one optical element behind each of the ports as well as another one in the center of the box. You can use a laser and a laser mount with a wheel (that you can put marks on it with a pencil) in order to rotate the laser.

Equipment (see also Fig. 4)

A Black-box

B Laser module in mount with wheel (the same laser module is used for both experiments, to be placed on the table surface)

C Transparent block

D Masking tape, pencil, ruler, paper with diagonal scale

Do not look directly into the laser beam, and make sure no other people are hit by it. Do not look into the optical ports of the box if the laser is on, and turn the laser off when not needed.

Task E2.1 - Central element (~0.3 pts)

The two optical axes cross in the center of the black box. At the crossing could sit: no element, a fully reflective mirror (both sides), a semi-transparent mirror, or a regular-triangle-shaped prism.

Determine which element is placed centrally in the black box. Describe its orientation towards the optical ports (A, B, C, D) - for example by using a sketch. Justify your choices.

Task E2.2 - Elements in remaining slots (~2.2 pts)

There is one element from Table 1 in each of the four slots behind the optical ports A, B, C, D, respectively.

Determine for each slot the type of element present. Justify your choices.

Task E2.3 - Properties (~7.5 pts)

In Table 1, you can find a second column containing characteristic properties of the possible elements.

Determine these characteristic properties for the optical elements used inside the box at slots A, B, C and D **as precisely as possible**.

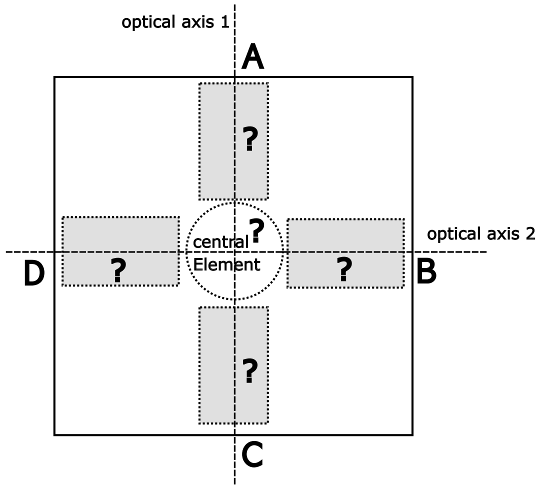

Figure 2: Layout of the black box and the slots of the unknown elements

Table 1: Possible elements in the slots of the black-box

no element	there is just air in the slot
mirror	angle between the mirror axis and one of the optical axes
prism	angle between one of its sides and one of the optical axes of the black box, shaped like regular triangle
concave or convex lens	distance to the center of the box, magnitude and sign of the focal length. *Note: Axes of lenses are always along optical axes.*
polarizer	angle of orientation relative to the vertical axis of the black-box
single, thin slit	distance to the center of the box, width of the slit
diffraction grating	distance to the center of the box, direction of stripes, distance between the stripes
pin hole	distance to the center of the box, hole diameter

Important hints:

- The wavelength of the laser is (650 ± 5) nm.
- The refractive index of transparent elements can be assumed to be 1.5.

Pictures of experimental setups and equipment

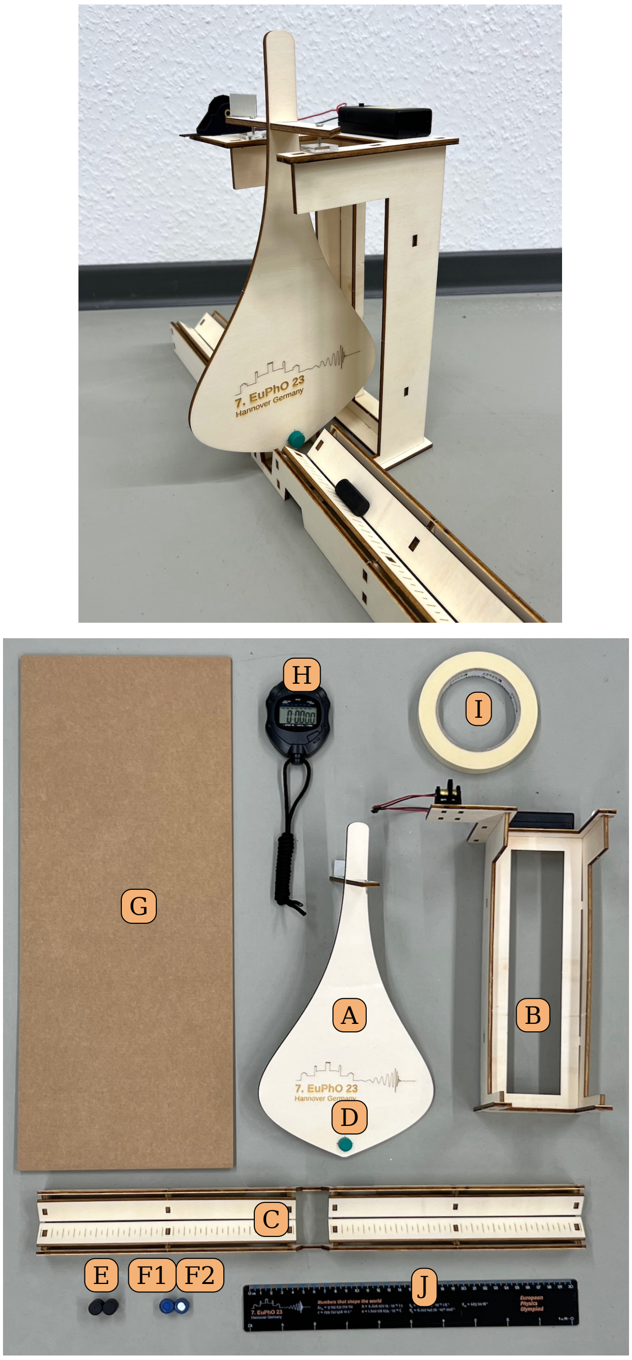

Figure 3: Setup and equipment for experimental problem E1.

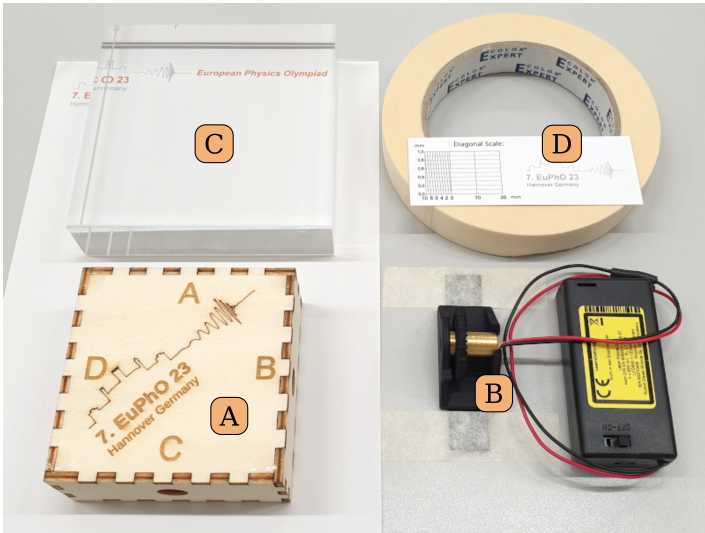

Figure 4: Setup and equipment for experimental problem E2. *Note: You can secure the laser module to the table using the tape - see C.*

Note: The laser module is initially mounted to the setup for E1. You have to take it off so you can use it for E2. You can also put it back (pay attention to alignment) when you want to switch back to E1.